

Preface

This is the book of proceedings of the Second International Workshop on Quantum Nonstationary Systems (QNS), held at the International Center of Physics (ICP) of University of Brasilia from August 28 to September 1, 2023. This hybrid format meeting was the second edition of a conference that occurred in 2009, at that time organized by the professors Viktor V. Dodonov (University of Brasilia, Brasilia, Brazil), Vladimir I. Man'ko (Lebedev Physics Institute, Moscow, Russia) and Salomon S. Mizrahi (Federal University of São Carlos, São Carlos, Brazil). Following the success of the first edition, the second QNS workshop gathered in Brasilia leading researchers and students in the field of quantum nonstationary phenomena, creating an environment where the latest achievements in the area, which comprises Quantum Information, Quantum Optics and cold atoms theory, could be divulged and debated. The event, organized by Alexandre Dodonov, Caio C. H. Ribeiro and Olavo L. S. F., also seized the opportunity to celebrate the 75-th birthday and the retirement of Prof. Viktor V. Dodonov from University of Brasilia.

The book contains 19 chapters. Chapter 1 gives a brief history and plans for the future of the Workshops on Quantum Nonstationary Systems (with photos of participants), while the remaining 18 chapters contain original works authored by the invited speakers:

- Chapter 2: V. V. Dodonov and A. Dodonov from University of Brasilia, Brazil
- Chapter 3: S. K. Suslov from Arizona State University, USA
- Chapter 4: J. Tito Mendonça from Universidade de Lisboa, Portugal
- Chapter 5: V. I. Yukalov and E. P. Yukalova from Joint Institute for Nuclear Research, Russia and Universidade de São Paulo, Brazil
- Chapter 6: D. Valente from Universidade Federal de Mato Grosso, Brazil
- Chapter 7: A. Vourdas from University of Bradford, UK
- Chapter 8: S. S. Mizrahi from Federal University of São Carlos, Brazil
- Chapter 9: J. P. Gazeau from Université Paris Cité, France
- Chapter 10: Olavo L. S. F. from University of Brasilia, Brazil

- Chapter 11: T. Mihaescu and A. Isar from National Institute of Physics and Nuclear Engineering and University of Bucharest, Romania
- Chapter 12: A. Marinho and A. Dodonov from Universidade Federal Rural da Amazônia and University of Brasilia, Brazil
- Chapter 13: S. N. Belolipetskiy, V. N. Chernega, V. I. Grebenkin and O. V. Man'ko from Bauman Moscow State Technical University, Russian University of Transport and Lebedev Physical Institute, Russia
- Chapter 14: G. Wilson and B. M. Garraway from University of Sussex, UK
- Chapter 15: C. C. Holanda Ribeiro from University of Brasilia, Brazil
- Chapter 16: M. A. Man'ko and V. I. Man'ko from Lebedev Physical Institute, Russia
- Chapter 17: E. P. Glasbrenner, Y. Gerdes, S. Varró and W. P. Schleich from Universität Ulm, Germany, ELI-ALPS Research Institute, Hungary and Texas A&M University, USA
- Chapter 18: G. de Oliveira and L. C. Céleri from Federal University of Goias, Brazil
- Chapter 19: B. Goren, K. K. Barley and S. K. Suslov from Arizona State University and Howard University, USA

The talks presented at the conference were stored online on the Youtube channel of the International Center of Physics at www.youtube.com/@cifunb. In addition to the technical presentations, the workshop also featured two talks with some historical accounts about the life and career of Prof. Viktor V. Dodonov, as the talk by Prof. Dodonov himself about his 50 years working with Quantum Mechanics (www.youtube.com/watch?v=1WSd1Rk0qGE) and the presentation by Prof. Salomon S. Mizrahi about the move of the Dodonov family to Brazil (www.youtube.com/watch?v=4trR7Yf8pBQ). Additional information about the venue, participants and the program of the event is available at the website <https://is.gd/2ndQNS>.

Alexandre Dodonov and Caio Cesar Holanda Ribeiro
Brasilia – DF – Brazil
May 12, 2024

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