Chapter 15

An exact solution for the quantum backreaction in a Bose-Einstein condensate

Caio C. Holanda Ribeiro

Institute of Physics and International Center of Physics,
University of Brasilia, Brasilia, DF, Brazil

E-mail: caio.ribeiro@unb.br

15.1 Introduction

The phenomenon of Bose-Einstein condensation is certainly one of the most intriguing ones in the modern sciences. Since it was predicted in the last century [1], a great deal of interesting applications and nuances involving these condensates were unveiled [2]. We call attention, for instance, to the recent simulation and observation of the Hawking Radiation in a condensate operating as an analog black hole [3], an effect of essentially impossible detection for astrophysical black holes. Yet, although experimentation with condensates have reached an unprecedented level of control, there are still a few theoretical aspects not completely understood for those systems.

To quote an example, if the particles in a condensate are allowed to interact, then not all particles of the gas are in the condensed phase, a phenomenon called quantum depletion. The latter was predicted by Bogoliubov in 1947 [4] and it was observed for the first time in [5] with a technique based on the distinct spatial distribution of the depleted cloud with respect to the condensed cloud. However, it was shown in [6] that the method employed in [5] most likely detected not only depletion, but also part of the condensed cloud, and the reason for it is that as the depleted cloud evolves, the condensate gets corrected due to backreaction.

Another nuance linked to the interpretation of the experiments is the difficulty in assessing the condensate initial condition from a given experiment. In particular, interacting condensates undergo phase diffusion [7, 8], which is the spontaneous destruction of the condensate off-diagonal long-range order and prevents the existence of finite size stationary condensates. Accordingly, it is in general not possible to determine the instantaneous state of a condensate from a given experimental realization of an interacting gas without knowing the full history of the system.

Due to its fundamental relevance to our understanding of Bose-Einstein condensates (BEC), in this work we present an exact solution for the evolution of a condensate in a toroidal configuration that can be realized with current technology. In order to address the phase diffusion and the problem of determining the condensate instantaneous state, we adopt the strategy of [6] and let the gas enter an interacting regime starting from a non-interacting configuration in which phase diffusion is negligible. We show that even for this simplified, homogeneous configuration, the rate in which the particles leave the condensate is a non-monotonic function of time.

The work is organized as follows. In section 15.2 we review the backreaction analysis developed in [6, 9] and present the field equations which govern the evolution of the condensate. The homogeneous condensate in a toroidal setup is shown in section 15.3 as well as the quantization of the theory. Section 15.4 contains the main results of our analysis, and we finish with a few final remarks in section 15.5.

15.2 Field equations

In this section we review, following the same notation, the backreaction equations developed in [6]. Let us consider a non-relativistic bosonic gas whose particles are allowed to move in only one spatial dimension. A cold gas in this setup was used, for instance, to probe the analogue Hawking radiation [3]. We assume that the gas is described by the field $\Psi = \Psi(t, x)$, subjected to the field equation

$$i\partial_t \Psi = \left(-\frac{\partial_x^2}{2m} + U + g|\Psi|^2 \right) \Psi, \tag{15.1}$$

where we have set $\hbar = 1$, m is a parameter, and U is the external (trapping) potential. Straightforward manipulations of the above equation then show that *any* solution Ψ gives rise to the conservation law $\partial_t \rho + \partial_x J = 0$, where $\rho = |\Psi|^2$, $J = (\Psi^* \partial_x \Psi - \Psi \partial_x \Psi^*)/2im$ are the system particle and current densities, respectively. Thus, the total number of particles, $N = \int dx \rho$, is conserved throughout the system evolution.

In principle, Eq. (15.1) describes the system evolution in its entirety, including possible quantum effects. However, the problem of finding a solution Ψ is remarkably convoluted even for simple setups and in general it is not possible to distinguish quantum from classical effects by inspecting Ψ alone. In order to circumvent these difficulties, we adopt an "improved" (Bogoliubov) expansion of the field variable that distinguishes the "quantum" part of the system evolution while maintaining the particle number conservation. Specifically, we assume that $N \to \infty$ and expand Ψ in powers of N as

$$\Psi = \phi_0 + \chi + \zeta + \mathcal{O}(N^{-3/2}), \tag{15.2}$$

with the following set of scaling behaviors [9]

$$N \to \infty \quad \text{with} \quad Ng = \text{constant}, \tag{15.3a}$$

$$\phi_0 = \mathcal{O}(N^{1/2}), \chi = \mathcal{O}(N^0), \text{ and } \zeta = \mathcal{O}(N^{-1/2}). \tag{15.3b}$$

The first condition (15.3a) is necessary to control the potential $g|\Psi|^2$ in the field equation (15.1), for when $N \to \infty$ for a fixed coupling g, $\Psi \to 0$ is the only physical solution to the field equation. The second condition (15.3b) ensures that $|\zeta| \ll |\chi| \ll |\phi_0|$ and identifies the different magnitude scales in the system. The field χ is the fluctuating field and ζ encapsulates the "backreaction" effects from χ onto the given condensate configuration ϕ_0. Returning the expansion (15.2) into the field equation (15.1) and identifying terms according to their order in N (keeping in mind that $g \propto N^{-1}$), we obtain to leading order the Gross-Pitaevskii equation for ϕ_0

$$i\partial_t \phi_0 = \left(-\frac{\partial_x^2}{2m} + U + g\rho_0\right)\phi_0, \tag{15.4}$$

$\rho_0 = |\phi_0|^2$, in the next order in N the Bogoliubov-de Gennes (BdG) equation for χ

$$i\partial_t \chi = \left(-\frac{\partial_x^2}{2m} + U + 2g\rho_0\right)\chi + g\phi_0^2\chi^*, \tag{15.5}$$

and finally a BdG-like equation with χ-dependent source terms for the final contribution to the expansion, ζ,

$$i\partial_t \zeta = \left(-\frac{\partial_x^2}{2m} + U + 2g\rho_0\right)\zeta + g\phi_0^2\zeta^* + 2g|\chi|^2\phi_0 + g\chi^2\phi_0^*. \tag{15.6}$$

In what follows we consider number-conservation of the theory up to order N^0, i.e., the densities ρ and J should be expanded also up to order N^0. This is the reason why we should keep ζ in the expansion, as both ρ and J are quadratic in Ψ.

Following [9] we can also define the auxiliary field $\phi_\mathrm{c} = \phi_0 + \zeta$. Both equations (15.4) and (15.6) can be combined and expressed in terms of ϕ_c via the improved Gross-Pitaevskii equation, which includes subleading terms

$$i\partial_t \phi_\mathrm{c} = \left(-\frac{\partial_x^2}{2m} + U + g|\phi_\mathrm{c}|^2 + 2g|\chi|^2\right)\phi_\mathrm{c} + g\chi^2\phi_\mathrm{c}^*. \tag{15.7}$$

The fields ϕ_c and χ suggest that we interpret ζ as modeling corrections to the condensate order parameter ϕ_0 due to the field χ. We will show in the following that this interpretation of the field ζ facilitates the proper formulation of backreaction of quantum fluctuations onto the classical background.

15.2.1 Quantization

In order to study quantum fluctuations we promote the classical field χ to the operator-valued distribution $\hat{\chi}$ and look for representations of the equal-time bosonic commutation relation $[\hat{\chi}(t,x), \hat{\chi}^\dagger(t,x')] = \delta(x - x')$. In this work we consider a (quasiparticle) vacuum state for the fluctuations, for which $\langle\hat{\chi}\rangle = 0$. After quantization, the classical current and density become operator-valued distributions as well, and for the vacuum state under consideration one has

$$\rho := \langle\hat{\rho}\rangle = |\phi_\mathrm{c}|^2 + \langle\hat{\chi}^\dagger\hat{\chi}\rangle + \mathcal{O}(N^{-1/2}), \tag{15.8}$$

$$J := \langle\hat{J}\rangle = \frac{1}{m}\mathrm{Im}\left[\phi_\mathrm{c}^*\partial_x\phi_\mathrm{c} + \langle\hat{\chi}^\dagger\partial_x\hat{\chi}\rangle\right] + \mathcal{O}(N^{-1/2}). \tag{15.9}$$

Furthermore, the field ϕ_c is now given by the operator-valued version of equation (15.7), where it appears as a multiple of the identity operator. Thus the equation coincides with its vacuum expectation value, and is given by

$$i\partial_t\phi_c = \left[-\frac{\partial_x^2}{2m} + U + g|\phi_c|^2 + 2g\langle\hat{\chi}^\dagger\hat{\chi}\rangle\right]\phi_c + g\langle\hat{\chi}^2\rangle\phi_c^*, \tag{15.10}$$

where the normal ordering prescription was taken.

To make the distinction between condensate and non-condensed cloud sharper, we reserve the nomenclatures of quantum depletion and phonon flux to the averaged contributions $\rho_\chi := \langle\hat{\chi}^\dagger\hat{\chi}\rangle$ and $J_\chi := \mathrm{Im}\,[\langle\hat{\chi}^\dagger\partial_x\hat{\chi}\rangle]/m$, respectively. Therefore, $\rho := \rho_c + \rho_\chi + \mathcal{O}(N^{-1/2})$ and $J := J_c + J_\chi + \mathcal{O}(N^{-1/2})$, and we obtain from Eqs. (15.10) and (15.5) that

$$\partial_t\rho_c + \partial_x J_c = ig(\phi_c^2\langle\hat{\chi}^{\dagger 2}\rangle - \phi_c^{*2}\langle\hat{\chi}^2\rangle), \tag{15.11a}$$
$$\partial_t\rho_\chi + \partial_x J_\chi = -ig(\phi_c^2\langle\hat{\chi}^{\dagger 2}\rangle - \phi_c^{*2}\langle\hat{\chi}^2\rangle), \tag{15.11b}$$

thus ensuring that the theory is conserving ($\partial_t\rho + \partial_x J = 0$) up to our working ($N^0$) order in the densities and currents.

15.2.2 Corrections to the condensate background

From the definitions of $\phi_c = \phi_0 + \zeta$ and ρ_c, J_c from Eqs. (15.8) and (15.9), we find that $\rho_c = \rho_0 + \rho_\zeta + \mathcal{O}(N^{-1/2})$ and $J_c = J_0 + J_\zeta + \mathcal{O}(N^{-1/2})$, where $J_0 = \mathrm{Im}[\phi_0^*\partial_x\phi_0]/m$,

$$\rho_\zeta = 2\mathrm{Re}\,[\phi_0^*\zeta], \tag{15.12}$$
$$J_\zeta = \frac{1}{m}\mathrm{Im}\,[\phi_0^*\partial_x\zeta + (\partial_x\phi_0)\zeta^*]. \tag{15.13}$$

It thus follows from Eq. (15.4) that $\partial_t\rho_c + \partial_x J_c = \partial_t\rho_\zeta + \partial_x J_\zeta = \mathcal{O}(N^0)$, ensuring the consistency of Eq. (15.11a) up to our working order.

The "density" ρ_ζ and "current density" J_ζ are the corrections to the condensate contributions ρ_0 and J_0. Within the validity domain of Bogoliubov theory i.e., when $\delta N := \int dx\rho_\chi \ll N = \int dx\rho_0$, the quantum fluctuations modeled by the field $\hat{\chi}$ remain small and independent of the condensate corrections ζ, which are in turn determined by ρ_χ and $\langle\hat{\chi}^2\rangle$ through Eq. (15.6). In this regime, the dynamics of the field $\hat{\chi}$ is linear. Furthermore, the very condensate existence *in the presence of interactions* ($\delta N/N \ll 1$) leads to a nonvanishing quantum depletion ρ_χ and nonvanishing anomalous correlator $\langle\hat{\chi}^2\rangle$, which in turn correct the condensate via the field ζ. This is the essence of the adopted backreaction scheme.

15.3 Condensates in a toroidal configuration

Let us solve the backreaction equations to a class of condensates of great experimental relevance: homogeneous condensates trapped in a toroidal configuration. We note that in [9] the authors showed how the condensate density is corrected for a 3D gas in the thermodynamic limit. However, in actual experiments the thermodynamic limit is not realizable, and phase diffusion plays a prominent role. We consider a trapping potential in Eq. (15.1) for which Ψ is defined for $-\ell/2 < x \leq \ell/2$, and

$$\Psi|_{x=-\ell/2} = \Psi|_{x=\ell/2}, \tag{15.14}$$

$$\partial_x\Psi|_{x=-\ell/2} = \partial_x\Psi|_{x=\ell/2}. \tag{15.15}$$

The above boundary conditions are "naturally" imposed by the field equation. Accordingly, because of their independence, the fields ϕ_0, χ, and ζ are also subjected to the same conditions.

A homogeneous condensate is thus obtained by assuming a constant potential U and a background order parameter $\phi_0 = \sqrt{\rho_0}\exp(-i\mu t)$, for constant ρ_0, such that the GP equation (15.4) reduces to an equation for the chemical potential μ:

$$\mu = U + g\rho_0. \tag{15.16}$$

Furthermore, in order to avoid the indeterminacy of the quasiparticle vacuum [10], we assume that for $t < 0$ the condensate is in a non-interacting regime ($g = 0$), and at $t = 0$ the interactions are suddenly turned on: $g = g_0 > 0$. Thus, $\mu = U \equiv \mu_0$ for $t < 0$ and $\mu = U + g_0\rho_0 \equiv \mu_0 + \Delta\mu$, where $\Delta\mu = g_0\rho_0$.

We call attention to the spatial translation invariance of this condensate realization, which implies that there are no particle fluxes during the condensate evolution, i.e., $J_0 = J_\chi = J_\zeta = 0$, and the continuity equation reduces to $\partial_t(\rho_\chi + \rho_\zeta) = 0$. Accordingly, $\rho_\zeta = -\rho_\chi$, and the condensate evolution is determined by the quantum depletion alone. We stress that this property does not occur for more complex condensate configurations.

15.3.1 Field modes

The background condensate ϕ_0 just presented allows for the construction of an exact expression for the quantum depletion. We start by defining a new variable $\psi(t, x) = \exp(i\mu t)\chi(t, x)$, such that Eq. (15.5) reduces to

$$i\partial_t\psi = -\frac{\partial_x^2\psi}{2m} + g\rho_0(\psi + \psi^*). \tag{15.17}$$

The solutions of this equation are easily found in terms of the Nambu spinor $\Phi = (\psi, \psi^*)^T$, where T stands for the matrix transpose operation. The spinor Φ satisfies

$$\frac{i}{\Delta\mu}\sigma_3\partial_t\Phi = -\frac{\xi_0^2\partial_x^2\Phi}{2} + \frac{g}{g_0}(1 + \sigma_1)\Phi, \tag{15.18}$$

where $\xi_0 = 1/\sqrt{mg_0\rho_0}$ is the condensate healing length in the interacting regime and σ_i, $i = 1, 2, 3$, are the Pauli matrices. Moreover, only the solutions Φ of Eq. (15.18) that satisfy $\Phi = \sigma_1\Phi^*$ correspond to solutions of Eq. (15.17), and for any two such solutions, say Φ and Φ', (15.18) implies that the scalar product

$$\langle\Phi, \Phi'\rangle = \int dx\Phi^\dagger\sigma_3\Phi' \tag{15.19}$$

is time-independent, and can be used to normalize the field modes. If $\{\Phi_n\}_n$ is a complete set of (positive norm) solutions such that $\langle\Phi_n, \Phi_{n'}\rangle = \delta_{n,n'}$, then the general solution of

Eq. (15.18) subjected to the reflection property is given by

$$\Phi(t,x)=\sum_n[a_n\Phi_n(t,x)+a_n^*\sigma_1\Phi_n^*(t,x)]. \tag{15.20}$$

In order to find the complete set of positive norm field modes, we note first that due to the spatial translation symmetry we can look for solutions in the form $\Phi(t,x)=\exp(ikx)\Phi_k(t)$. Then by imposing the (toroidal) periodic boundary conditions, we find that $\exp(-ik\ell/2)=\exp(ik\ell/2)$, or $\sin(k\ell/2)=0$, from which we obtain that

$$k\equiv k_n=\frac{2n\pi}{\ell},\quad n\text{ integer}. \tag{15.21}$$

Now for each n, let us determine $\Phi_{k_n}(t)$. In the non-interacting regime $(t<0, g=0)$, solutions in the form $\Phi_{k_n}(t)=\exp(-i\omega_n t)\Phi^0_{k_n}$, for constant $\Phi^0_{k_n}$ and $\omega_n>0$ exist. In fact, Eq. (15.18) fixes

$$\omega_n=\frac{\xi_0^2\Delta\mu}{2}k_n^2,\quad \Phi^0_{k_n}=(1,0)^T, \tag{15.22}$$

and

$$\Phi_n(t,x)=\frac{1}{\sqrt{\ell}}e^{-i\omega_n t+ik_n x}(1,0)^T, \tag{15.23}$$

for integer n, exhausts all positive norm field modes in the non-interacting regime. Our goal now is to determine how each $\Phi_n(t,x)$ evolves into the interacting regime $(g>0)$, and this can be done in a straightforward manner by using the method of [6], namely, we expand $\Phi_n(t,x)$ in a complete set of field modes in the interacting regime and use the continuity in time at $t=0$ to fix the solution. We find that

$$\Phi_n(t,x)=\frac{e^{ik_n x}}{\sqrt{\ell}}\left[e^{-i\Delta\mu\Omega_n t}\frac{\xi_0^2k_n^2/2+\Omega_n}{2\Omega_n\xi_0^2k_n^2}(\xi_0^2k_n^2/2+\Omega_n,\xi_0^2k_n^2/2-\Omega_n)^T+(\Omega_n\leftrightarrow-\Omega_n)\right], \tag{15.24}$$

for $n\neq 0$ with

$$\Omega_n=\xi_0|k_n|\left(1+\frac{\xi_0^2k_n^2}{4}\right)^{1/2}, \tag{15.25}$$

and

$$\Phi_0(t,x)=\frac{1}{\sqrt{\ell}}(1-i\Delta\mu t, i\Delta\mu t)^T, \tag{15.26}$$

for $n=0$. Finally, quantization is achieved by promoting each Fourier coefficient a_n in Eq. (15.20) to an operator $\hat{a}_n$ subjected to the canonical commutation relation $[\hat{a}_n,\hat{a}^\dagger_{n'}]=\delta_{n,n'}$. Furthermore, the vacuum state, $|0\rangle$, is defined by the kernel condition $\hat{a}_n|0\rangle=0$ for all n. Thus,

$$\hat{\chi}(t,x)=e^{-i\mu t}\sum_n\left[\hat{a}_n\Phi_{n,1}(t,x)+\hat{a}_n^\dagger\Phi^*_{n,2}(t,x)\right] \tag{15.27}$$

is the operator-valued distribution that models the quantum fluctuations on top of the background condensate.

15.4 Quantum depletion

Once the quantum field expansion (15.27) is known, the quantum depletion, ρ_χ, is readily found to be

$$\rho_\chi = \sum_n |\Phi_{n,2}(t,x)|^2 = \frac{(\Delta\mu t)^2}{\ell} + \frac{2}{\ell}\sum_{n=1}^{\infty} \frac{\sin^2(\Delta\mu\Omega_n t)}{\Omega_n^2}, \qquad (15.28)$$

where in the last equality we separated off the contribution from the zero modes ($n = 0$) of the sum. Notice that due to the system spatial translation invariance the quantum depletion is independent of the spatial coordinate x, and thus for this system we can work with the total number of depleted particles $\delta N = \int dx\rho_\chi = \ell\rho_\chi$ without loss of generality. Note also that there is only one free parameter, namely, the ratio $\beta = \xi_0/\ell$, in δN. Indeed, the chemical potential $\Delta\mu$ always appears multiplied by t and can thus be absorbed via a change of units, whereas in Eq. (15.25) we find $\xi_0 k_n = \beta(2n\pi)$. Therefore, we present our results in terms of β, that measures how extended is the condensate is terms of its healing length.

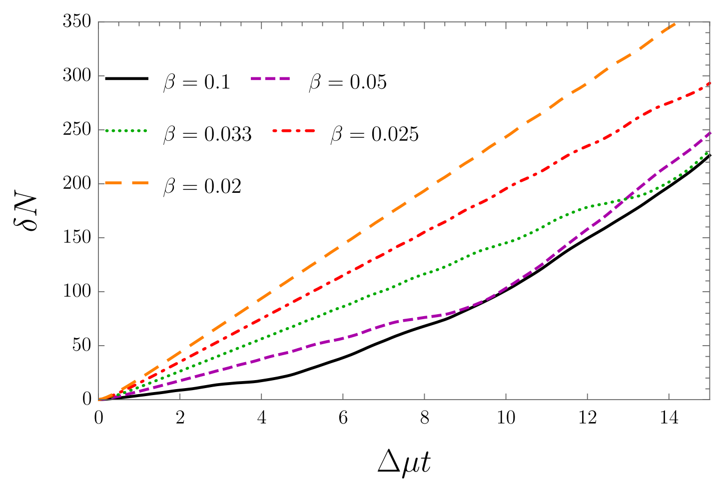

Figure 15.1: Number of depleted particles for several values of β. As expected, the phase diffusion leads to a condensate degradation, i.e., δN increases as time passes. We note a curious effect hidden in these plots: the degradation rate, given by $d\delta N/dt$, is not a monotonic function.

Let us analyze the first term appearing in Eq. (15.28). This term does not depend on β and it comes from the zeros modes of the field $\hat{\chi}$. Accordingly, this contribution is exactly the one coming from the phase diffusion of the condensate. We also call attention to the fact that this term coincides with the one obtained in [6] for a rather different condensate configuration.

In Fig. 15.1 we depict plots of δN as function of time for several values of β. For definiteness, we assume that the expansion of section 15.2 holds true as long as $\delta N/N \leq 0.1$, i.e., at most ten percent of the particles are not in the condensate. Furthermore, typically a condensate can be formed with $N \sim 3000$, and thus for the profiles of Fig. 15.1 our expansion is reliable. The salient features observed in Fig. 15.1 include the fairly linear growth of δN with time for some plots, which should be compared to the t^2 dependence coming from the phase diffusion. Also, we note the intriguing behavior of the condensate degradation rate $d\delta N/dt$, which is not a monotonic function. Recall that there is no reason to expect a non-monotonic behavior of $d\delta N/dt$ for such a homogeneous condensate, showing how rich the backreaction analysis can be. In fact, we see that during their evolution a condensate with $\beta = 0.033$ is more preserved than the one with $\beta = 0.025$ at about $t\Delta\mu \sim 14$, which is a rather a counter-intuitive effect because $\delta N \to \infty$ as $\beta \to 0$. In Fig. 15.2 we depict the degradation rate for $\beta = 0.1, 0.05$ and 0.02, where the the

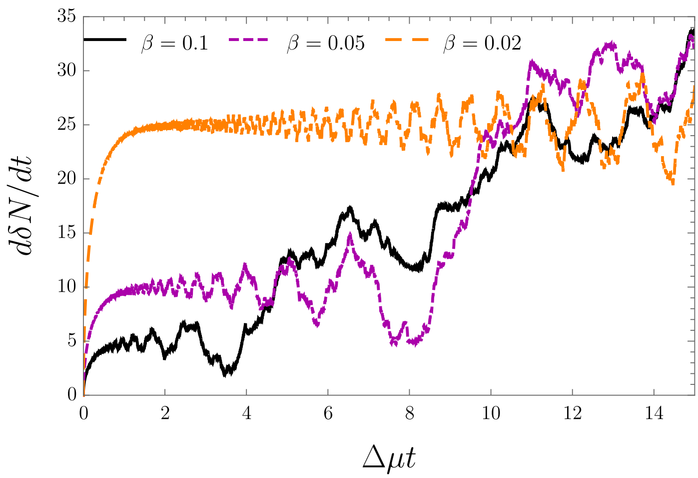

Figure 15.2: The degradation rate $d\delta N/dt$ as function of time for $\beta = 0.1, 0.05$ and 0.02. We observe that the condensate degradation is highly complex even for this simplified condensate configuration, and includes regimes of acceleration and retardation.

oscillation between the acceleration and retardation of the degradation rate is manifest. Furthermore, we see from the plots of Fig. 15.2 that as the interactions are turned on ($t = 0$), the depletion cloud initially develops in an accelerated manner before reaching a linear regime.

15.5 Final Remarks

The concept of backreaction in quantum field theories is notoriously convoluted and rewarding. Here we have reviewed the backreaction analysis developed in [9] for Bose-Einstein condensates and the method was applied for a condensate trapped in a toroidal configuration that can be readily produced with current technology. Among our findings, we have shown that even for a homogeneous background condensate at rest with respect to the laboratory frame, the quantum fluctuations lead to a non-trivial condensate degradation where the depleted particle number has a non-monotonic rate. This result reveals the difficulty in tracking distinct physical phenomena in the backreaction analysis on top of a more complex condensate configuration. For instance, we call attention to the experiment of [3], where in addition to the condensate phase diffusion, one also has an effective event horizon, and the backreaction analysis remains, up to this day, an open problem.

We conclude this work with a final remark. It is important to stress that the scheme put forward in [9] and further elucidated [6] also considers the notion of a quantum force density. The latter vanishes, though, for the condensate we considered here because there are no particle fluxes. Nevertheless, both approaches are based on the same set of fundamental equations, given by Eqs. (15.4), (15.5), and (15.6).

Acknowledgments

The author would like to thank the International Center of Physics, the Institute of Physics of the University of Brasilia, and the Fundação de Apoio à Pesquisa do Distrito Federal (TOA 387/2023, process 00193-00002051/2023-14) for supporting this work.

15.6 References

[1] S. N. Bose, Plancks Gesetz und Lichtquantenhypothese, Z. Phys. **26**, 178 (1924); A. Einstein, Quanten theorie des einatomigen idealen gases, Sitz.ber. Kgl. Preuss. Akad. Wiss. **1924**, 261-267 (1924); **1925** 3-14, (1925).

[2] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Theory of Bose-Einstein condensation in trapped gases, Rev. Mod. Phys. **71**, 463 (1999).

[3] J. R. Muñoz de Nova, K. Golubkov, V. I. Kolobov, and J. Steinhauer, Observation of thermal Hawking radiation and its temperature in an analogue black hole, Nature (London) **569**, 688 (2019).

[4] N. N. Bogolyubov, On the theory of superfluidity, J. Phys. (USSR) **11**, 23-32 (1947).

[5] R. Lopes, C. Eigen, N. Navon, D. Clément, R. P. Smith, and Z. Hadzibabic, Quantum Depletion of a Homogeneous Bose- Einstein Condensate, Phys. Rev. Lett. **119**, 190404 (2017).

[6] S.-S. Baak, C. C. Holanda Ribeiro, and U. R. Fischer, Number-conserving solution for dynamical quantum backreaction in a Bose-Einstein condensate, Phys. Rev. A **106**, 053319 (2022).

[7] M. Lewenstein and L. You, Quantum Phase Diffusion of a Bose- Einstein Condensate, Phys. Rev. Lett. **77**, 3489 (1996).

[8] Y. Castin and R. Dum, Low-temperature Bose-Einstein condensates in time-dependent traps: Beyond the $U(1)$ symmetrybreaking approach, Phys. Rev. A **57**, 3008 (1998).

[9] R. Schützhold, M. Uhlmann, Y. Xu, and U. R. Fischer, Quantum backreaction in dilute Bose-Einstein condensates, Phys. Rev. D **72**, 105005 (2005).

[10] C. C. Holanda Ribeiro, S.-S. Baak, and U. R. Fischer, Existence of steady-state black hole analogs in finite quasi-one- dimensional Bose-Einstein condensates, Phys. Rev. D **105**, 124066 (2022).

Cite this work as:

C. C. H. Ribeiro, An exact solution for the quantum backreaction in a Bose-Einstein condensate, in A. Dodonov and C. C. H. Ribeiro (Eds.), Proceedings of the Second International Workshop on Quantum Nonstationary Systems, pp. 249–257 (LF Editorial, São Paulo, 2024). ISBN: 978-65-5563-446-4.