Bose-Einstein Condensates: concepts, experimental aspects, and applications

Caio C. Holanda Ribeiro

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INTERNATIONAL CENTER OF PHYSICS
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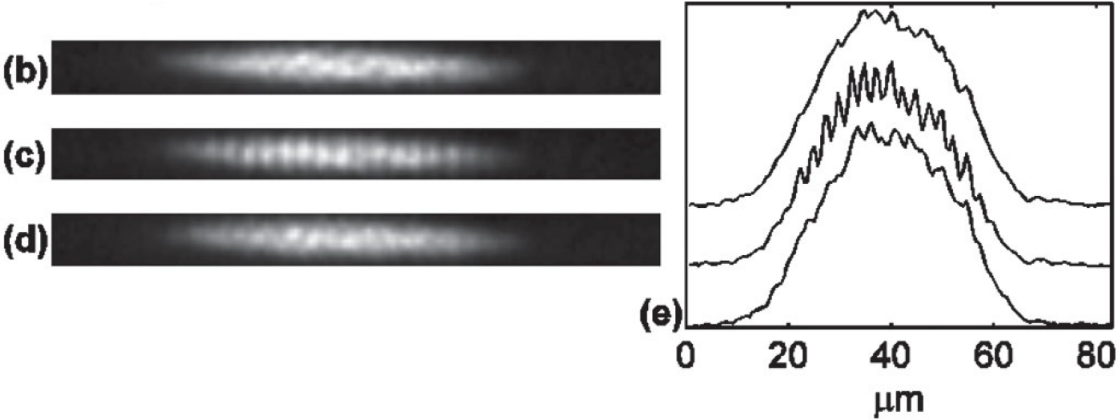
Part one: The phenomenon of condensation

Steinhauer et al, PRL **109**, 195301 (2012)



We consider the non-relativistic theory for ψ

$$S = \int \mathrm{d}^4 x \left[i\psi^* \partial_t \psi - i\psi \partial_t \psi^* - \frac{1}{2m} |\nabla \psi|^2 - \frac{V}{2} |\psi|^2 \right].$$

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$\Rightarrow -\partial_t \rho = \nabla \cdot \mathbf{J}$ on classical solutions, where

$\rho = |\psi|^2$ is the particle density

$\mathbf{J} = \frac{1}{2im}(\psi^* \nabla \psi - \psi \nabla \psi^*)$ is the particle current

$N = \int \mathrm{d}^3x \rho$ is the number of particles.

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Field variables: ψ, ψ^*. Canonically conjugated momentum:

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For a harmonic trap: $V = m(\omega_x x^2 + \omega_y y^2 + \omega_z z^2)/2$:

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- $\mathbf{n} = (n_x, n_y, n_z)$
- $\int \mathrm{d}^3 x \phi^*_{\mathbf{n}} \phi_{\mathbf{n}'} = \delta_{\mathbf{n},\mathbf{n}'}$
- $\omega_{\mathbf{n}} = \omega_x(n_x + 1/2) + \omega_y(n_y + 1/2) + \omega_z(n_z + 1/2)$.
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$$H = \int \mathrm{d}^3x(\pi\partial_t\psi + c.c.) - L = \sum_{\mathbf{n}} \omega_{\mathbf{n}} a^*_{\mathbf{n}} a_{\mathbf{n}}$$

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- Grand canonical ensemble:

$$\langle \hat{O} \rangle = \frac{1}{Z} \text{tr } \hat{O} e^{-\beta(\hat{H}-\mu\hat{N})}$$

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$$N = \langle\hat{N}\rangle = \frac{1}{\beta}\frac{\partial \ln Z}{\partial\mu} = \sum_{\mathbf{n}}\frac{1}{e^{\beta(\omega_{\mathbf{n}}-\mu)}-1} = N_0 + \sum_{\mathbf{n}\neq 0}\frac{1}{e^{\beta(\omega_{\mathbf{n}}-\mu)}-1}$$

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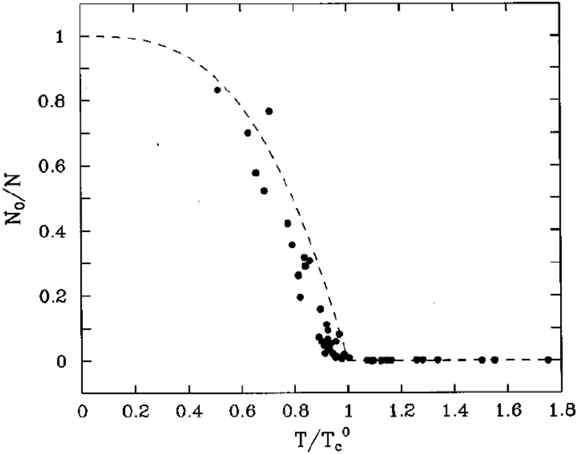
Typical values: $T_c = 60$ nK for $N = 1000$ and $\omega = 100$ Hz.

Dalfovo et al, Rev. Mod. Phys. **71**, 463 (1999)



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Typical values of N_0 are between 1000 and 5000 (why?)!

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Further considerations

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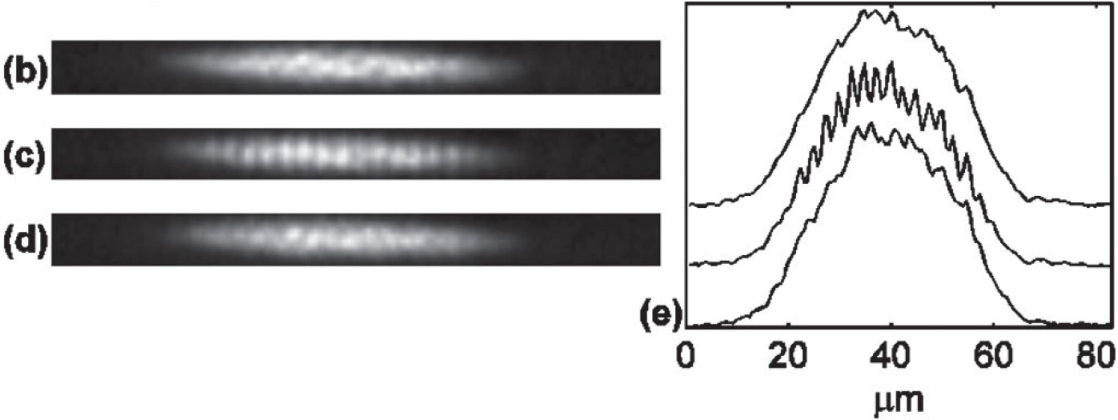
$$(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \left(V + g\rho - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right),$$

where $\mathbf{v} = \nabla\theta$ in the fluid velocity.

Part two: Condensate “environment”

Steinhauer et al, PRL **109**, 195301 (2012)



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- $\delta\hat{\psi}$ satisfies the Bogoliubov-de Gennes equation:

$$i\partial_t\delta\hat{\psi} = \left(-\frac{1}{2m}\nabla^2 + V + 2g|\psi_0|^2\right)\delta\hat{\psi} + g\psi_0^2\delta\hat{\psi}^\dagger$$

The particle densities become:

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- Bogoliubov expansion breaks the $U(1)$ symmetry: $-\partial_t \delta\rho \neq \nabla \cdot \delta\mathbf{J}$

PRL **119,** 190404 (2017) PHYSICAL REVIEW LETTERS week ending 10 NOVEMBER 2017

Quantum Depletion of a Homogeneous Bose-Einstein Condensate

Raphael Lopes,[1,*] Christoph Eigen,[1] Nir Navon,[1,2] David Clément,[3] Robert P. Smith,[1] and Zoran Hadzibabic[1]
[1]*Cavendish Laboratory, University of Cambridge, J.J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
[2]*Department of Physics, Yale University, New Haven, Connecticut 06511, USA*
[3]*Laboratoire Charles Fabry, Institut d'Optique Graduate School, CNRS, Université Paris-Saclay, 91127 Palaiseau cedex, France*

(Received 12 June 2017; published 7 November 2017)

We measure the quantum depletion of an interacting homogeneous Bose-Einstein condensate and confirm the 70-year-old theory of Bogoliubov. The observed condensate depletion is reversibly tunable by changing the strength of the interparticle interactions. Our atomic homogeneous condensate is produced in an optical-box trap, the interactions are tuned via a magnetic Feshbach resonance, and the condensed fraction is determined by momentum-selective two-photon Bragg scattering.

DOI: 10.1103/PhysRevLett.119.190404

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- $\langle \delta\hat{\theta}^2 \rangle$ diverges with t^2 in interacting condensates:

PHYSICAL REVIEW LETTERS

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Quantum Phase Diffusion of a Bose-Einstein Condensate

M. Lewenstein[1] and L. You[2]

[1] *Commissariat à l'Energie Atomique, DSM/DRECAM/SPAM, Centre d'Etudes de Saclay, 91191, Gif-sur-Yvette, France*
[2] *Institute for Theoretical Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, MS 14, Cambridge, Massachusetts 02138*

(Received 20 May 1996)

We discuss the quantum properties of the Bose-Einstein condensate of a dilute gas of atoms in a trap. We show that the phase of the condensate undergoes quantum diffusion which can be detected in far off-resonant light scattering experiments. [S0031-9007(96)01469-X]

PHYSICAL REVIEW A VOLUME 57, NUMBER 4 APRIL 1998

Low-temperature Bose-Einstein condensates in time-dependent traps: Beyond the $U(1)$ symmetry-breaking approach

Y. Castin and R. Dum
Ecole Normale Supérieure, Laboratoire Kastler Brossel, 24, Rue Lhomond, F-75231 Paris Cedex 05, France
(Received 25 February 1997; revised manuscript received 7 October 1997)

We present a method to calculate the dynamics of very-low-temperature Bose-Einstein condensates in time-dependent traps. We consider a system with a well-defined number of particles, rather than a system in a coherent state with a well-defined phase. This preserves the $U(1)$ symmetry of the problem. We use a systematic asymptotic expansion in the square root of the fraction of noncondensed particles. In lowest order we recover the time-dependent Gross-Pitaevskii equation for the condensate wave function. The next order gives the linear dynamics of noncondensed particles. The higher order gives corrections to the time-dependent Gross-Pitaevskii equation including the effects of noncondensed particles on the condensate. We compare this method with the Bogoliubov–de Gennes approach.
[S1050-2947(98)00604-0]

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S. M. Barnett: “The robust description of the condensate, therefore, is that of a coherent-like state undergoing phase diffusion.”

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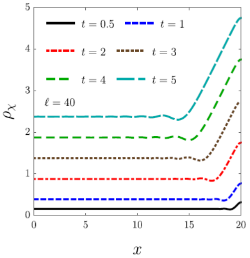
$$(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \left(V + g\rho - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right) + \frac{\mathbf{Q}}{\rho}$$

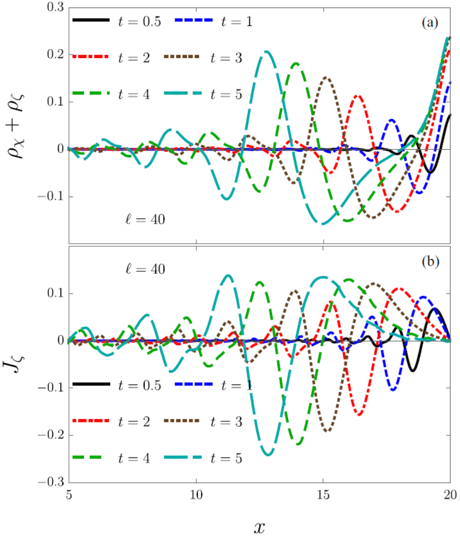
- Interactions lead to condensate degradation.
- Atoms may enter or leave the condensate
- The depleted cloud still interact with the condensate

Is the condensate evolution ruled by Euler equations?
We should consider quantum corrections!
Fischer et al, PRD **72**, 105005 (2005): $-\partial_t \rho = \nabla \cdot \mathbf{J}$ and

$$(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \left(V + g\rho - \frac{1}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right) + \frac{\mathbf{Q}}{\rho}$$

- $\mathbf{Q}$ depends only on $\delta\hat{\psi}$.





Baak et al, PRA **106**, 053319 (2022)

Part three: Analog Hawking radiation

Let us consider the problem of probing Hawking radiation.

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- $T_H \sim 10^{-8}$ K for a black hole of one solar mass

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PHYSICAL REVIEW LETTERS

VOLUME 46 | 25 MAY 1981 | NUMBER 21

Experimental Black-Hole Evaporation?

W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 8 December 1980)

It is shown that the same arguments which lead to black-hole evaporation also predict that a thermal spectrum of sound waves should be given out from the sonic horizon in transsonic fluid flow.

Dalfovo et al, Rev. Mod. Phys. **71**, 463 (1999):
"In the presence of harmonic confinement, the many-body theory of interacting Bose gases gives rise to several unexpected features. This opens new theoretical perspectives in this interdisciplinary field, where useful concepts coming from different areas of physics (atomic physics, quantum optics, statistical mechanics, and condensed-matter physics) are now merging together."

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Class. Quantum Grav. **15** (1998) 1767–1791. Printed in the UK PII: S0264-9381(98)90306-9

Acoustic black holes: horizons, ergospheres and Hawking radiation

Matt Visser†
Physics Department, Washington University, Saint Louis, MO 63130-4899, USA

Received 1 December 1997

Abstract. It is a deceptively simple question to ask how acoustic disturbances propagate in a non-homogeneous flowing fluid. Subject to suitable restrictions, this question can be answered by invoking the language of Lorentzian differential geometry. This paper begins with a pedagogical derivation of the following result: if the fluid is barotropic and inviscid, and the flow is irrotational (though possibly time dependent), then the equation of motion for the velocity potential describing a sound wave is identical to that for a minimally coupled massless scalar field propagating in a (3 + 1)-dimensional Lorentzian geometry

$$\Delta\psi \equiv \frac{1}{\sqrt{-g}}\partial_\mu \left(\sqrt{-g}\, g^{\mu\nu}\, \partial_\nu \psi\right) = 0.$$

Analog Hawking radiation

Dalfovo et al, Rev. Mod. Phys. **71**, 463 (1999): "In the presence of harmonic confinement, the many-body theory of interacting Bose gases gives rise to several unexpected features. This opens new theoretical perspectives in this interdisciplinary field, where useful concepts coming from different areas of physics (atomic physics, quantum optics, statistical mechanics, and condensed-matter physics) are now merging together."

Note that $T_c \sim T_H$!

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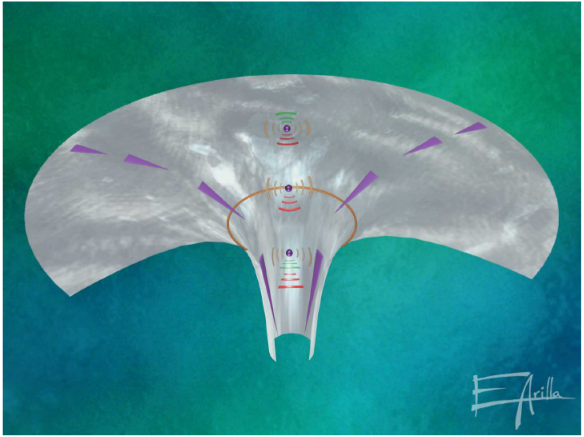
Analog Hawking radiation



For a generic BEC:

$$i\partial_t\hat{\psi} = \left(-\frac{1}{2m}\nabla^2 + V + g\hat{\psi}^\dagger\hat{\psi}\right)\hat{\psi}.$$

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Madelung:

$$\hat{\psi} = e^{i(\theta_0+\delta\hat{\theta})}\sqrt{\rho_0 + \delta\hat{\rho}}$$

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$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\delta\hat{\theta} = 0,$$

$$g_{\mu\nu} = \frac{\rho_0}{mc}\begin{pmatrix} \mathbf{v}^2 - c^2 & -v_x & -v_y & -v_z \\ -v_x & 1 & 0 & 0 \\ -v_y & 0 & 1 & 0 \\ -v_z & 0 & 0 & 1 \end{pmatrix}, c^2 = \frac{g\rho_0}{m}.$$

Suppose one can implement an analog black hole using a BEC.

Suppose one can implement an analog black hole using a BEC. How to probe the spontaneous radiation?

Suppose one can implement an analog black hole using a BEC. How to probe the spontaneous radiation?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **78**, 021603(R) (2008)

Nonlocal density correlations as a signature of Hawking radiation from acoustic black holes

Roberto Balbinot,[1] Alessandro Fabbri,[2] Serena Fagnocchi,[1,3] Alessio Recati,[4] and Iacopo Carusotto[4]

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[2]*Departamento de Fisica Teorica and IFIC, Universidad de Valencia-CSIC, C. Dr. Moliner 50, 46100 Burjassot, Spain*

[3]*Centro Studi e Ricerche "Enrico Fermi," Compendio Viminale, 00184 Roma, Italy*

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(Received 13 December 2007; published 19 August 2008)

We have used the analogy between gravitational systems and nonhomogeneous fluid flows to calculate the density-density correlation function of an atomic Bose-Einstein condensate in the presence of an acoustic black hole. The emission of correlated pairs of phonons by Hawking-like process results into a peculiar long-range density correlation. Quantitative estimations of the effect are provided for realistic experimental configurations.

The density-density correlation is defined as

$$G^{(2)}(x, x') = \langle \hat{\rho}(x)\hat{\rho}(x')\rangle - \langle \hat{\rho}(x)\rangle\langle \hat{\rho}(x')\rangle$$

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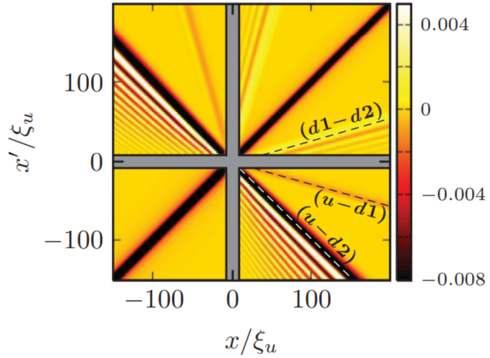
Pavloff et al, PRA **85**, 013621 (2012)

For an acoustic BH it should look like:



LETTER

https://doi.org/10.1038/s41586-019-1241-0

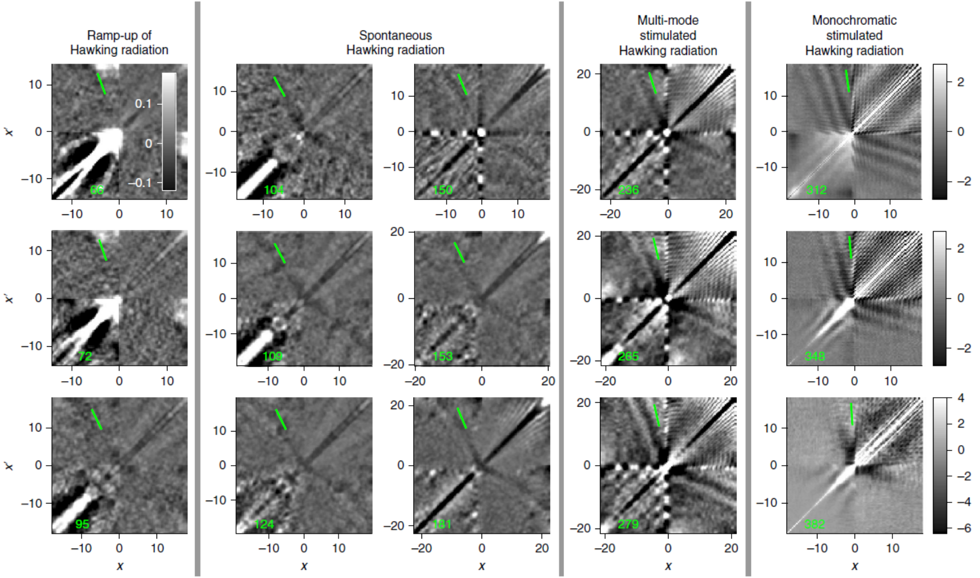
Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

The entropy of a black hole[1] and Hawking radiation[2] should have the same temperature given by the surface gravity, within a numerical factor of the order of unity. In addition, Hawking radiation should have a thermal spectrum, which creates an information paradox[3,4]. However, the thermality should be limited by greybody factors[5], at the very least[6]. It has been proposed that the physics of Hawking radiation could be verified in an analogue system[7], an idea that has been carefully studied and developed theoretically[8–18]. Classical white-hole analogues have been investigated experimentally[19–21], and other analogue systems have been presented[22,23]. The theoretical

a real black hole. In a Bose–Einstein condensate, the dispersion relation is linear in the low-energy limit. Thus, we should create an analogue black hole with sufficiently low Hawking temperature that the radiation is in the linear regime of the dispersion relation. We can then test whether the emitted Hawking radiation obeys the prediction of equation (1). There are several theoretical works suggesting that this should be the case. Using previous analytical results for a system similar to this experiment[13], we find that equation (1) gives an accurate prediction for $k_B T_H \lesssim 0.14\, mc_{out}^2$, where m is the mass of the atom. We will show that the experiment is within this limit. Parentani and colleagues[29] studied

Nature Physics, **17**, 362–367 (2021)



- Impact of dimension on the condensate dynamics: Hohenberg theorem
- Impact of dipolar interactions
- Condensate of composite bosons?
- Entanglement
- Measurement-induced phase diffusion

Thank You!

Questions?