Chapter 6

Quantum cloning in waveguide-QED inspired by nonequilibrium self-assembly

D. Valente

Institute of Physics, Universidade Federal de Mato Grosso,
Cuiabá, MT, Brazil

6.1 Introduction

When matter is driven away from thermal equilibrium, impressive things can happen. Fast oscillating mirrors can make photons pop out from the vacuum state of the electromagnetic field. Jiggling molecules can become assembled as parts of living cells. Both effects can be witnessed with current experiments: photons have been created from the dynamic modulation of an optical cavity [1], and synthetic cells are currently being developed [2]. But there seems to be something deeper with living entities, something that attracts the curiosity of non-biologists like Erwin Schrödinger [3], Alan Turing [4], John von Neumann [5], Ilya Prigogine [6], and Freeman Dyson [7], just to name a few. What is it?

Perhaps it has to do, at least in part, with the fact that a lot of human intention and engineering is required to build complex devices, tools, and experiments. Precise driving forces shall make mirrors oscillate in just the right way, if one wants to produce single photons by means of the dynamical Casimir effect. By contrast, whatever the sources of free energy that have pushed inanimate matter into living beings, be it hydrothermal vents or solar light, no human intention or ingenuity has been involved. The driving forces that first produced, and that keeps producing, living ensembles of matter have no encoded goals from the outset. There must be some significant physics we can learn from this.

Nonequilibrium self-assembly is one way of trying to make progress on this type of

quest. Equilibrium self-assembly is, of course, more familiar to physicists. Atomic order in crystals is well understood in terms of minimizing free energy. When a dissipative system is driven away from thermal equilibrium, and no particular goal is encoded in the driving forces, what happens? Most of the time, movements and positions may look random. But sometimes exceptional arrangements and patterns emerge. The system may eventually become self-organized in certain states and behaviors that remind us of what living things do.

Here, we argue that life-inspired nonequilibrium self-organization can, and should, be extended to the quantum regime. This could spur fresh thinking about sensing, communication, computation, and energetics, phenomena studied with living beings as well as with quantum technologies. Because living systems are far from thermal equilibrium and dissipative, we might imitate at least some of their successes with dissipative quantum systems as an alternative to only aiming at ever higher isolation.

6.2 Nonequilibrium self-assembly

6.2.1 Experiments: self-replicating spheres and self-assembling wires

Let us have a look at the "self-replicating spheres" experiment of the Self-Assembly Lab at the Massachusetts Institute of Technology (MIT). This movie shows a few spheres free to roll on top of a moving table; If you are reading the printed version, please visit the webpage `https://selfassemblylab.mit.edu/`. The spheres are soft, passive, contain magnetic beads inside them, but no biological or robotic components. Each sphere may either stick to or unstick from its neighbors, somewhat at random. The table provides the mechanical work driving the spheres out of equilibrium, with a driving pattern that is maintained throughout the experiment. As time goes by, and new spheres are added as raw material, we see the self-assembly of circular structures resembling a two-dimensional cell. The cell-like structure grows and divides, akin to biological cells. In Fig.6.1, we make a sketch of the process.

In the experiment discussed above, the division of the artificial cells is quite reversible and unstable. That is, two artificial cells may merge back into a single one almost as easily as it splits into two. This is unlike biological cells. The processes where replicated bacteria merge back into a single one is not to be expected. In much the same way, plants and animals do not ungrow in time [8]. This stability, or irreversibility, has to do with heat dissipation.

We illustrate how statistical irreversibility is related with energy dissipation with a second experiment. This video from Stanford University shows metallic beads immersed in a viscous fluid; If you are reading the printed version, please look for the movie entitled "self-assembling wires" at `https://www.youtube.com/@StanfordComplexity/videos`. The source of work now is a static voltage applied to the system, with a tip coming from the top. A metallic ring encircling metallic beads is kept at ground voltage. The spheres then self-assemble into wires, allowing electric current to flow through the system. The filaments follow the tip when it moves, as if the system was intentionally seeking the energy source that maintains its worm-like state [9]. The wires are formed in a viscous fluid, while the spheres are dragged by the air. This is key to explain the higher stability

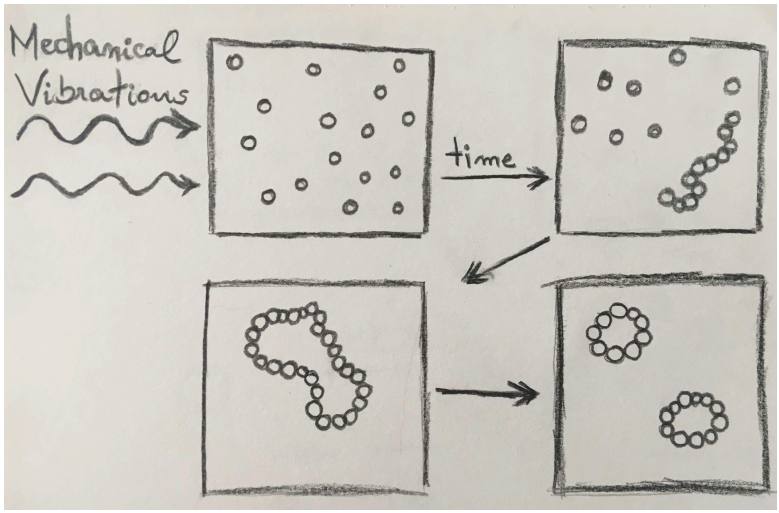

Figure 6.1: Sketch of the "self-replicating spheres" experiment conducted at MIT (see the movie at `https://selfassemblylab.mit.edu/`). Mechanical vibrations on a moving table provide the work source for the soft spheres to stick or unstick, apparently at random. With time, the spheres eventually self-organize in a two-dimensional cell-like structure, and also divide into two cells.

of the self-assembled wires as compared with the self-replicating spheres.

6.2.2 Theory: dissipative adaptation

Dissipative adaptation is a hypothetical thermodynamic principle to describe nonequilibrium self-assembly. It has been coined by the physicist Jeremy England [8, 10], in tribute to Prigogine's concept of dissipative structures [6]. England's idea was to generalize Boltzmann's distribution, which links the thermal equilibrium probability p_k of a system to occupy a given physical state k to the energy E_k of that state as

$$p_k \propto e^{-\beta E_k}, \tag{6.1}$$

where $\beta = (k_B T)^{-1}$, given in terms of the temperature T, and the Boltzmann constant k_B.

By following a stochastic-thermodynamics way of thinking, specially Jarzynski [11] and Crooks [12] fluctuation theorems, England emphasizes that the history of a particle from an initial state i to a final state j is most important in nonequilibrium scenarios. The transition probability from i to j, namely

$$p_{i \to j}, \tag{6.2}$$

should now be the key variable biased by some energetic constraint. However, when dynamical transitions come into play, energy exchanges between the system and its environment start playing a role. It should thus be reasonable that the work W performed on the system and the heat it dissipates affect the transition probabilities. The intuition is that, provided enough work, the system can jump unexpectedly high barriers, reaching

otherwise atypical states. Because conservative systems could simply jump back those atypically high barriers, the presence of heat dissipation to the environment creates the mechanism for stabilization of the system in the final state, hence introducing statistical irreversibility. This should explain how, given a certain drive, a system can dissipatively adapt to the patterns of its nonequilibrium environment.

In mathematical terms, the dissipative adaptation reads [10]

$$\frac{p_{i\to j}}{p_{i\to k}} = e^{-\beta E_{kj}} \frac{p^*_{j\to i}}{p^*_{k\to i}} \frac{\langle e^{-\beta W}\rangle_{ik}}{\langle e^{-\beta W}\rangle_{ij}}, \tag{6.3}$$

showing how both equilibrium (Boltzmann) and nonequilibrium (kinetic and thermodynamic) aspects compete in determining the configurations of the system. The kinetic term is described by the time-reversed transition probability $p^*_{j\to i}$. The analogy with the barrier hight is again useful here. Regardless of how much work the system absorbs and dissipates, the system tends to occupy more frequently the states that are reversibly more accessible due to low barriers. Finally, the thermodynamic dependence is given by the absorbed work that appears in $\langle e^{-\beta W}\rangle_{ij}$, with the average taken with fixed initial i and final j states. The bigger the absorbed work W from i to j, the higher the probability of that transition to occur.

6.3 Quantum dissipative adaptation

Does the notion of dissipative adaptation survive at the quantum regime? Mathematically speaking, Eq.(6.3) clearly diverges at the zero-temperature limit, where quantum effects are most pronounced. This means we cannot apply it directly to the quantum domain. Also, we may guess that quantum coherent effects could imply a violation of the dissipative adaptation hypothesis. Classically, work is required for a system to jump over energy barriers in order to achieve exceptional states. Quantum coherence, such as tunneling for instance, could provide a mechanism of organization with no jump over barriers, possibly with no work cost, while dissipation would guarantee its stability. Finally, it is not obvious how quantum fluctuations in the source of energy would affect the adaptation of the system, so the very definition of work would be in question in such a scenario.

6.3.1 Our main result

As it turns out, a clear connection between average work and transition probability can be found in the extreme quantum regime of zero temperature, with the energy input provided by single-photon pulses. Most interestingly, our results are valid under conditions where Crooks' and Jarzynski's theorems cannot be applied, suggesting a deeper foundation for the dissipative adaptation hypothesis. Our model assumes a system-plus-environment approach, where single-photon pulses drive single three-level atoms in Λ configuration, each atom dissipatively coupled to a thermal bath at temperature $T = 0$. Our quantum dissipative adaptation reads [13]

$$p_{a\to b} = \frac{\Gamma_b}{\Gamma_a + \Gamma_b} \frac{\langle W\rangle_a}{\hbar\omega_a}, \tag{6.4}$$

where $|a\rangle$ and $|b\rangle$ correspond to the two lowest (and equally stable) energy levels of the Λ systems. Here, $\hbar\omega_a$ is the energy difference between the state $|a\rangle$ and the excited state $|e\rangle$.

The spontaneous emission rates, denoted by Γ_a and Γ_b, describe the kinetic contribution to our quantum dissipative adaptation. Finally, $\langle W \rangle_a$ is the average work performed by a single-photon pulse on a single Λ atom during the transition $|a\rangle \to |b\rangle$. In Fig.6.2, we picture the concept behind our quantum dissipative adaptation.

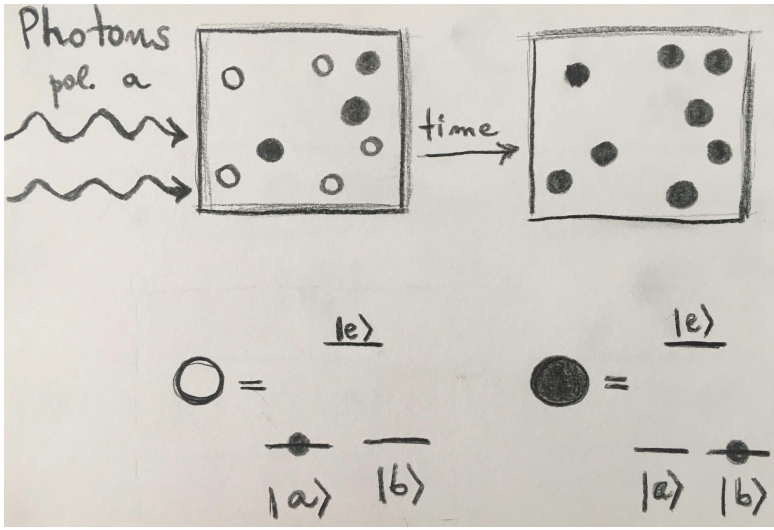

Figure 6.2: Sketch of the model revealing a quantum dissipative adaptation, as described by Eq.6.4. Polarized single-photon pulses hit an ensemble of atoms in a mixture of states $|a\rangle$ (light circle) and $|b\rangle$ (full circle). The atoms have three energy levels in Λ configuration, where $|a\rangle$ and $|b\rangle$ are the lowest levels (not necessarily degenerate), and $|e\rangle$ is the excited state. As time goes by, the work $\langle W \rangle_a$ provided by the photons may leave the ensemble organized in a pure state $|b\rangle$, with a probability $p_{a\to b} \propto \langle W \rangle_a$. To compute the work, we consider a normalized pulse $|1_a\rangle$ of finite linewidth. The probabilistic nature of the dynamics is quantum at its origin, since the temperature is $T = 0$.

As we will see in more detail below, maximal work absorption does not mean maximal excitation probability. That is to say, quantum coherence allows the system to somehow "jump below the barrier" while still consuming work from the source (the photon, in that case). Quite on the opposite, the absorbed work $\langle W \rangle_a$ hits its maximum precisely when the excitation probability $p_{a\to e}$ hits its minimum, showing how the work cost of self-organization in the presence of quantum-coherent transitions is far from intuitive.

6.3.2 Experimental context of the quantum model: waveguide-QED

The model that led us to Eq.(6.4) refers to the experimental context known as waveguide quantum electrodynamics (waveguide-QED). In cavity-QED, photons get trapped between mirrors. In waveguide-QED, by contrast, single-photon pulses propagate in waveguides and strongly interact with matter, driving it out of equilibrium. Relaxation, encompassing decoherence, spontaneous emission, and stimulated emission, also takes

place through the guided modes [14–19].

One-dimensional (1D) waveguides can be implemented with semiconducting materials, such as nanophotonic wires, photonic crystals, and plasmonic nanowires. Few-level quantum systems are obtained with semiconducting quantum dots, basically nano-sized islands where electric dipoles (electron-hole pairs) are spatially confined. See review [20].

Alternatively, superconducting rings can also play the role of artificial atoms. The electric current becomes quantized, as reminiscent of Bohr's atomic model. Superconducting transmission lines are used as waveguides in this case [21].

6.3.3 The quantum model: single-photon pulses driving Λ atoms

We consider an electromagnetic environment with two orthogonal polarization modes, a_ω and b_ω, where ω are field frequencies. We consider the continuum limit of an unstructured environment, i.e., we assume a constant density of 1D modes $\rho_{1\mathrm{D}}$, so that,

$$\sum_\omega \to \int d\omega\ \rho_{1\mathrm{D}}(\omega) = \rho_{1\mathrm{D}} \int d\omega. \tag{6.5}$$

Initially, at time $t \to -\infty$, the electromagnetic field is in the vacuum state $|0\rangle$ for all the modes, corresponding to an environment in thermal equilibrium at temperature $T = 0$.

At time $t = 0$, a single-photon pulse is added to the atomic environment, with a well-defined polarization,

$$|1_a\rangle = \sum_\omega \phi^a_\omega(0) a^\dagger_\omega |0\rangle. \tag{6.6}$$

The pulse admits a space-dependent description, namely,

$$\phi_a(z,t) = \sum_\omega \phi^a_\omega(t) \exp(ik_\omega z), \tag{6.7}$$

where $k_\omega = \omega/c$, and c is the speed of light. If, for instance, the pulse has been prepared by the spontaneous emission of a distant two-level atom (not considered in the model), then $\phi_a(z,0)$ will take an exponential envelope profile.

We consider a dipole interaction with the three-level atom, in the rotating-wave approximation regime,

$$H_I = -i\hbar \sum_\omega g_a a_\omega \sigma^\dagger_a + g_b b_\omega \sigma^\dagger_b + \mathrm{h.c.}, \tag{6.8}$$

where $\sigma_a = |a\rangle\langle e|$, and $\sigma_b = |b\rangle\langle e|$. The states $|a\rangle$, $|b\rangle$ and $|e\rangle$ have been defined above. *h.c.* stands for hermitian conjugate. The couplings to the environment give rise to the spontaneous emission rates

$$\Gamma_{a,b} = 2\pi g^2_{a,b}\ \rho_{1\mathrm{D}}, \tag{6.9}$$

evidencing how dissipation emerges in our model.

The transition probability between the two stable atomic states $|a\rangle$ and $|b\rangle$ are given by standard quantum theory, namely,

$$p_{a\to b}(t) = \langle b|\mathrm{tr}_E[U|a,1_a\rangle\langle a,1_a|U^\dagger]|b\rangle, \tag{6.10}$$

where tr_E is the trace over the electromagnetic environment. Whenever $p_{a\to b}(\infty) = 1$, the atom achieves self-organization in the sense that any initial mixed state $\rho_S(0) =$

$p_a^{(0)}|a\rangle\langle a| + p_b^{(0)}|b\rangle\langle b|$ ends up at $\rho_S(\infty) = |b\rangle\langle b|$. In that case, the initial mixture present in $\rho_S(0)$ is transferred to the quantum state of the field, namely $\rho_E = \mathrm{tr}_S[\rho]$, where ρ is the state of the system-plus-environment universe. To achieve such ideal self-organization, the photon must be resonant with the atomic frequency $\omega_a = (E_e - E_a)/\hbar$. The transition frequency to state $|b\rangle$, i.e., $\omega_b = (E_e - E_b)/\hbar$, does not affect our results, which means that the final state can have either higher or lower energy without altering the probability transition.

In Eq.(6.10), the unitary operator corresponds to

$$U = \exp\left(-iHt/\hbar\right), \tag{6.11}$$

where the Hamiltonian describes the atom-plus-field universe,

$$H = H_S + H_I + H_E. \tag{6.12}$$

The atomic system is described by

$$H_S = \hbar\omega_a|e\rangle\langle e| + \hbar\delta_{ab}|b\rangle\langle b|, \tag{6.13}$$

so that $\delta_{ab} = \omega_a - \omega_b$. Finally, the environment is described by

$$H_E = \sum_\omega \hbar\omega(a_\omega^\dagger a_\omega + b_\omega^\dagger b_\omega), \tag{6.14}$$

where the $\sum_\omega \hbar\omega/2$ terms have been omitted for convenience. More details can be found in Ref.[13].

6.3.4 Work performed by a single-photon pulse on a single atom

We have defined work in two equivalent ways for this single-photon pulse model, one in the Schrödinger picture and the other in the Heisenberg picture (see proof of equivalence in Ref.[22]). Because the Heisenberg picture is more appealing to our classical intuition, it is worth highlighting it here. When a time-varying classical electric field $E(t)$ acts on a classical dipole $D(t) = qx(t)$, the work is given by $W = \int F\dot{x}\ dt = \int qE\dot{x}\ dt = \int \dot{D}E\ dt$. Analogously, we define the average work performed by a single-photon pulse on a quantum dipole as

$$\langle W\rangle = \int_0^\infty \langle(\partial_t D(t))\, E_{\mathrm{in}}(t)\rangle\ dt. \tag{6.15}$$

Only the incoming field, $E_{\mathrm{in}}(t) = \sum_\omega i\epsilon_\omega a_\omega e^{-i\omega t} + \mathrm{h.c.}$, is used in our definition, since the field produced by the atom that acts back on the atom itself gives rise to heat dissipation in our model. Operator $D(t)$ is the dipole operator, computed as $D(t) = U^\dagger D U$, with $D = \sum D_{ea}\sigma_a + \mathrm{h.c.}$. The constants are related with to coupling energy, $\hbar g_a = D_{ea}\epsilon_{\omega_a}$. The index for $\langle W\rangle_a$ in Eq.(6.4) states that the particular atomic initial state $|a\rangle$ is being assumed in the calculation of the average work.

6.3.5 Some analytical results

Our main result, Eq.(6.4), is valid for any pulse shape. To provide an analytically solvable example, we can think of an exponential pulse (as it would have been provided by the spontaneous emission from a hypothetical far-away two-level atomic source),

$$\phi_a(z,0) = N\Theta(-z)\exp[(\Delta/2 + i\omega_L)z/c], \tag{6.16}$$

where $N = \sqrt{2\pi\rho_{1D}}$ is a normalization constant, $\Theta(z)$ is the Heaviside step function, Δ is the spectral linewidth, ω_L is the central frequency, and c is the speed of light.

In that case, we find that

$$p_{a\to e}(t) \leq 4\Delta\Gamma_a/(\Gamma_a+\Gamma_b)^2. \tag{6.17}$$

This means that

$$p_{a\to e}(t) \to 0 \text{ when } \Delta \to 0, \tag{6.18}$$

corresponding to the monochromatic limit. In other words, the excitation probability vanishes in the monochromatic limit. However, the monochromatic regime also implies maximal self-organization,

$$p_{a\to b}(\infty) = \frac{\Gamma_b}{\Gamma_a+\Gamma_b}\frac{4\Gamma_a}{\Gamma_a+\Gamma_b+\Delta} \overset{\Delta\to 0}{=} \frac{4\Gamma_a\Gamma_b}{(\Gamma_a+\Gamma_b)^2} = 1, \tag{6.19}$$

provided that $\Gamma_a = \Gamma_b$.

The absorbed work also achieves maximal values in the monochromatic regime,

$$\langle W\rangle_a = \hbar\omega_a \frac{4\Gamma_a}{\Gamma_a+\Gamma_b+\Delta} \overset{\Delta\to 0}{=} 2\hbar\omega_a \quad (\text{if } \Gamma_a = \Gamma_b), \tag{6.20}$$

evidencing that an absorption-plus-emission picture is inadequate to characterize the (rather quantum coherent) work transfer.

6.3.6 Statistical irreversibility in the quantum model

We can now understand why the zero-temperature regime causes a divergence in the classical theory. When computing the reverse (backward) probability, with a pulse inverted in time, we find that

$$p^*_{b\to a}(t) = 0, \tag{6.21}$$

since the atom is transparent to the reversed driving pulse. Eq.(6.21) is valid under any pulse shape. In Crooks theorem, it is assumed that $p_{a\to b}/p^*_{b\to a} = \exp(\beta Q)$, where Q is the stochastic heat dissipated to the environment [12]. Here, both the left-hand and right-hand sides of this relation would diverge for a finite Q and zero temperature $\beta \to \infty$.

6.4 Quantum cloning

How close to the self-replicating spheres experiment discussed above can our quantum version of the dissipative adaptation take us?

6.4.1 Model

Let us consider a one-dimensional chain of Λ systems, each at equilibrium at temperature $T = 0$ in one of the two stable states $|a\rangle$ or $|b\rangle$, so that its quantum state reads

$$|a\rangle|a\rangle|b\rangle...|a\rangle|b\rangle. \tag{6.22}$$

To replicate this state, we need a second chain of Λ systems, playing the role of raw material. That is, this second chain should be in an "empty" (bare) state, containing no special information, let's say,

$$|b\rangle|b\rangle|b\rangle...|b\rangle|b\rangle. \tag{6.23}$$

Ideally, we hope to find a cloning transformation where

$$|b\rangle|b\rangle|b\rangle...|b\rangle|b\rangle \rightarrow |a\rangle|a\rangle|b\rangle...|a\rangle|b\rangle. \tag{6.24}$$

Note that there is no contradiction with the no-cloning theorem, which forbids the perfect copying of arbitrary superpositions (though imperfect copies can nevertheless be obtained), but allows basis states to be in principle perfectly cloned [16, 22, 23].

Thermodynamically, the single-photon pulses provide the energy source (the work) for this quantum cloning process. As this source is consumed, the field ends up in a modified state, so the full view of the cloning process here is more rigorously characterized as

$$U|\text{chain}_1\rangle|\text{chain}_2\rangle|\text{photons}\rangle \rightarrow |\text{chain}_1\rangle|\text{chain}_2'\rangle|\text{photons}'\rangle. \tag{6.25}$$

Ideally, we would like to find the emergence of $|\text{chain}_2'\rangle = |\text{chain}_1\rangle$, without having to finely engineer the photon state (i.e., spontaneously emitted photons should be enough).

To achieve that, we have to devise some interaction between the two chains. If each Λ system from the first chain pairs with its respective neighbor from the second chain, let's say, by means of some type of dipole-dipole interaction that is sensitive to the state of the atom, this would affect the dynamics of this empty chain when it is driven out of equilibrium by single photon pulses (now polarized in the b modes). For instance, let us take an atom-atom Hamiltonian in the form

$$H_{12} = -J\ \sigma_{a1}^{(z)}\sigma_{a2}^{(z)}, \tag{6.26}$$

where $\sigma_{a1}^{(z)} = |e_1\rangle\langle e_1| - |a_1\rangle\langle a_1|$ corresponds to one atom operator in the first chain, whose state is to be replicated, and $\sigma_{a2}^{(z)}$ describes the equivalent for one atom in the second (empty) chain. The coupling strength J alters the resonance of the photon with one atom conditional to the state of the other atom. As it may have been noticed, H_{12} is a dispersive-type interaction inspired by the Ising model for ferromagnets. Curiously, in the self-replicating spheres experiment magnetic dipoles are also employed so as to make the spheres interact. If that is mere coincidence or reveals something deeper, we do not know. In Fig.6.3, we illustrate the idea of our self-organized quantum cloning.

6.4.2 Results

As we show in Ref.[22], the probability for a successful replication in a single pair of atoms at times $t \rightarrow \infty$ (much longer than the single-photon pulse duration) is given by

$$p_{a1,b2\rightarrow a2,a2}(\infty) = P(\Delta, \delta_{L-bJ}), \tag{6.27}$$

where we have defined

$$P(\Delta, \delta_{L-bJ}) \equiv \frac{\Gamma^2}{\left(\frac{2\Gamma-\Delta}{2}\right)^2 + \delta_{L-bJ}^2}\left[1 + \frac{\Delta}{2\Gamma} - \frac{\Delta(2\Gamma+\Delta)}{\left(\frac{2\Gamma+\Delta}{2}\right)^2 + \delta_{L-bJ}^2}\right]. \tag{6.28}$$

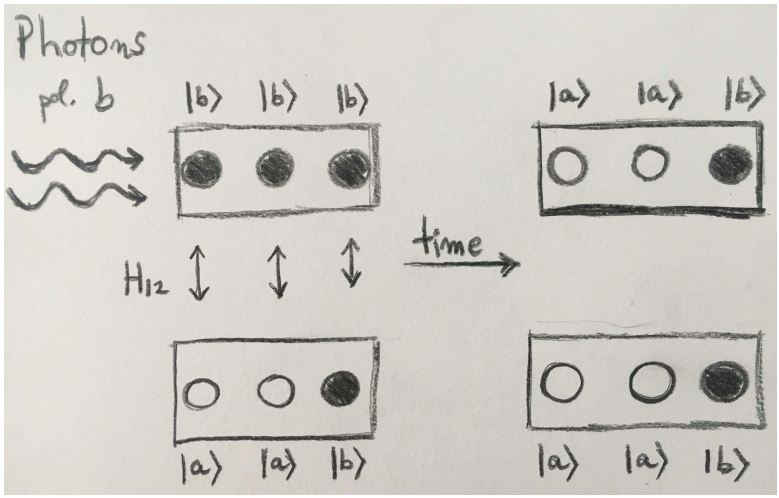

Figure 6.3: Sketch of the self-organized quantum cloning proposed in Eqs.(6.24) and (6.25). Photons polarized in b provide the work $\langle W\rangle_{a1,b2}$ for copying the states of the atoms in chain 1, $|a\rangle|a\rangle|b\rangle$, into the atoms of chain 2. Chain 2 undergoes transformation $|b\rangle|b\rangle|b\rangle \to |a\rangle|a\rangle|b\rangle$ in time, while keeping the original (template) chain 1 unaltered. The pairwise coupling between the chains, H_{12} as in Eq.(6.26), is responsible for static shifts in the atomic energy levels, thus consuming no work for that. The probability of a copying transition here is proportional to the work provided by the single-photon pulses, $p_{a1,b2\to a2,a2} \propto \langle W\rangle_{a1,b2}$, fulfilling the quantum dissipative adaptation discussed above.

We have again chosen the ideal scenario where $\Gamma = \Gamma_a = \Gamma_b$. We have also assumed an exponentially decaying pulse in time with spectral linewidth Δ, as in Eq.(6.16).

Here, the crucial parameter is the light-matter detuning, as affected by the dipole interaction strength J,

$$\delta_{L-bJ} = \omega_L - (\omega_b + J). \tag{6.29}$$

To see that, we can assume in Eq.(6.28) that a highly monochromatic photon ($\Delta \to 0$) arrives at resonance with the atomic frequency as given by $\omega_b + J$. In that case, we can in principle achieve perfect quantum state replication, since

$$p_{a1,b2\to a2,a2}(\infty) = P(0,0) = 1. \tag{6.30}$$

Again, the quantum dissipative adaptation is also fulfilled in this slightly more complicated case (with general detunings and finite atom-atom interactions),

$$p_{a1,b2\to a2,a2}(\infty) = \frac{\langle W\rangle_{a1,b2}}{2\hbar\omega_L}, \tag{6.31}$$

but now with the photon central frequency ω_L appearing instead of the atomic frequency (see Eq.(6.4) for comparison).

6.4.3 A word on mutations

Ideally, we hope that a perfect "dormant" transition would occur when the atom to be copied is at state $|b\rangle$. The probability for this dormant transition reads

$$p_{b1,b2\to b2,b2}(\infty) = 1 - P(\Delta, \delta_{L-b}). \tag{6.32}$$

That is, we take the same expression from Eq.(6.28), except that we now replace the detuning by

$$\delta_{L-b} = \omega_L - \omega_b, \tag{6.33}$$

which is given by the frequency of the atom undisturbed by the interaction strength J.

The interesting thing is that, for a photon of frequency $\omega_L = \omega_b + J$ (achieving perfect replication), the unperturbed detuning is given by $\delta_{L-b} = J$, which is finite ($J < \infty$) in any physically relevant scenario. This implies that random "mutations" during the replication of the chain are unavoidable, as introduced by the far-from-resonance driving photons themselves. It is surprising that such a simplistic model contains some analogy to ultraviolet-induced mutations in the genetic material of living cells. Further exploiting these random mutations to achieve some degree of evolutionary process, provided that some selection mechanism is present, sounds a promising line of research.

6.5 Conclusions

In summary, we have investigated the notion of dissipative adaptation as a thermodynamic principle underlying some types of nonequilibrium phenomena. In particular, it allowed us to draw a formal analogy between a classical theory of nonequilibrium self-assembling systems and a quantum model of a self-organizing system such as the three-level quantum system electromagnetically driven by a propagating single-photon pulse, as shown by Eqs. (6.3) and (6.4). As an application, we have conceived a quantum cloning process in a dissipative system driven out of equilibrium. This has been inspired by self-replication, a life-like behavior also underlying the experiment with soft spheres mechanically driven by an oscillating table. The work cost of our quantum cloning process has been found consistent with the dissipative adaptation hypothesis, as shown in Eq.(6.31).

In perspective, we would like to understand how broadly applicable is the idea of dissipative adaptation, with an eye to alternative implementations of quantum information processing in nonequilibrium dissipative systems. How are our results modified when two photons arrive simultaneously, thus inducing stimulated emission? Does the quantum dissipative adaptation play any role in photosynthesis? Does it hold for a quantum Brownian particle dissipatively tunneling in a bistable potential? Can an evolutionary-like processes emerge in our quantum cloning model? Could all of these contribute to new sensing, cryptography, and other quantum technologies? Can we find self-organized behaviors, therefore going beyond self-organized quantum states of dissipative quantum systems? As an encouraging first step towards this last question, we mention that the energy-seeking behavior appearing in the self-assembling wires [9] has been recently extrapolated to the quantum regime [24], thus revealing emergent functionalities such as adaptive heat control, comprised of self-organized energy gradients and active equilibration.

According to Albert Szent-Györgyi, Nobel-prize winner in physiology or medicine of 1937, "life is nothing but an electron looking for a place to rest". This provocative

statement suggests to us that our nonstationary quantum electronic devices are somewhat alive already. Can we make them even more alive? We would like to find it out.

Acknowledgments

D. V. thanks the organizers of the Second International Workshop on Quantum Nonstationary Systems for the kind invitation, and congratulates Prof Viktor Dodonov for his 75th birthday. D. V. also acknowledges Instituto Serrapilheira (grant no. Serra-1912-32056), and the INCT-IQ-CNPq (465469/2014-0).

6.6 References

[1] P. Lähteenmäki, G. S. Paraoanu, J. Hassel, and P. J. Hakonen, Dynamical Casimir effect in a Josephson metamaterial, Proc. Nat. Acad. Sci. **110**, 4234 (2013).
[2] Y. Elani and J. M. Seddon, What it means to be alive: a synthetic cell perspective, R. Soc. Int. Focus **13**, 5 (2023).
[3] E. Schrödinger, What is Life? The physical aspect of the living cell, Cambridge University Press, 1944.
[4] A. Turing, The chemical basis of morphogenesis, Phil. Tr. R. Soc. London B **237**, 37 (1952).
[5] J. von Neumann, Theory of self-reproducing automata, University of Illinois Press, 1966.
[6] G. Nicolis, and I. Prigogine, Self-organization in nonequilibrium systems: from dissipative structures to order through fluctuations, Wiley, 1977.
[7] F. Dyson, Origins of Life, Cambridge University Press, 1999.
[8] J. England, Statistical physics of self-replication, J. Chem. Phys. **139**, 121923 (2013).
[9] D. Kondepudi, B. Kay, and J. Dixon, End-directed evolution and the emergence of energy-seeking behavior in a complex system, Phys. Rev. E **91**, 050902(R) (2015).
[10] J. England, Dissipative adaptation in driven self-assembly, Nat. Nanotech. **10**, 919 (2015).
[11] C. Jarzynski, Nonequilibrium equality for free energy differences, Phys. Rev. Lett. **78**, 2690 (1997).
[12] G. E. Crooks, Entropy production fluctuation theorem and the nonequilibrium work relation for free energy differences, Phys. Rev. E **60**, 2721 (1999).
[13] D. Valente, F. Brito, and T. Werlang, Quantum dissipative adaptation, Comm. Phys. **4**, 11 (2021).
[14] D. Valente, S. Portolan, G. Nogues, J.-P. Poizat, M. Richard, J.-M. Gérard, M. F. Santos, A. Auffèves, Monitoring stimulated emission at the single-photon level in one-dimensional atoms, Phys. Rev. A, **85**, 023811 (2012).
[15] D. Valente, Y. Li, J.-P. Poizat, J.-M. Gérard, L. C. Kwek, M. F. Santos, and A. Auffèves, Optimal irreversible stimulated emission, New J. Phys., **14**, 083029 (2012).
[16] D. Valente, Y. Li, J.-P. Poizat, J.-M. Gérard, L. Kwek, M. Santos, and A. Auffèves, Universal optimal broadband photon cloning and entanglement creation in one-dimensional atoms, Phys. Rev. A, **86**, 022333 (2012).
[17] F. Fratini, E. Mascarenhas, L. Safari, J-Ph. Poizat, D. Valente, A. Auffèves, D. Ger-

ace, and M. F. Santos, Fabry-Perot Interferometer with quantum mirrors: nonlinear light transport and rectification, Phys. Rev. Lett. **113**, 243601 (2014).

[18] D. Valente, M. F. Z. Arruda, and T. Werlang, Non-Markovianity induced by a single-photon wave packet in a one-dimensional waveguide, Opt. Lett. **41**, 3126 (2016).

[19] N. Almeida, T. Werlang, and D. Valente, Mach-Zehnder interferometer with quantum beamsplitters, J. Opt. Soc. Am. B **36**, 3357 (2019).

[20] P. Lodahl, S. Mahmoodian, and S. Stobbe, Interfacing single photons and single quantum dots with photonic nanostructures, Rev. Mod. Phys. **87**, 347 (2015).

[21] O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Y. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai, Resonance fluorescence of a single artificial atom, Science **327**, 840 (2010).

[22] D. Valente, Self-replication of a quantum artificial organism driven by single-photon pulses, Sci. Rep. **11**, 16433 (2021).

[23] W. K. Wootters, and W. H. Zurek, A single quantum cannot be cloned, Nature **299**, 802 (1982).

[24] T. Werlang, M. Matos, F. Brito, and D. Valente, Emergence of energy-avoiding and energy-seeking behaviors in nonequilibrium dissipative quantum systems, Comm. Phys. **5**, 7 (2022).

Cite this work as:

D. Valente, Quantum cloning in waveguide-QED inspired by nonequilibrium self-assembly, in A. Dodonov and C. C. H. Ribeiro (Eds.), Proceedings of the Second International Workshop on Quantum Nonstationary Systems, pp. 99–111 (LF Editorial, São Paulo, 2024). ISBN: 978-65-5563-446-4.